

UNIVERSITY OF PITTSBURGH

DEPARTMENT OF MECHANICAL ENGINEERING

NASA RESEARCH GRANT NsG 634

DYNAMIC AND STATIC ANALYSES OF STRUCTURES WITH
UNIFORMLY DISTRIBUTED AXIAL LOADS

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Interim Progress Report

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I. General

Under the terms of the original grant, this project was to terminate May 1, 1965. However, an extension was granted extending the completion date to September 1, 1965. Sufficient funds are in the budget to cover this extension.

However, since the granting of this extension, the personnel assignments have changed. In October, 1964, Mr. Sanwar Verma was hospitalized and subsequently withdrew from the University. His position on the project was filled as of January 1, 1965, by Mr. Charles E. Jamison. Mr. Jamison received his B.S. degree in Mechanical Engineering from the University of West Virginia in January, 1965. He is currently a Graduate Research Assistant in this department.

II. Closed Form Solution

During the first six months of the project, much effort was spent attempting to find a closed form solution of the governing differential equation in terms of known functions. The ordinary differential equation under consideration may be written in the form

$$\frac{d^4 y}{dx^4} + x \frac{d^2 y}{dx^2} + \frac{dy}{dx} - \lambda^4 y = 0 \quad (1)$$

The following attempts at a solution were unsuccessfully tried:

- a. A Variation of Parameters approach in terms of Hyperbolic, Trigonometric, Bessel, and Airy Functions.
- b. The differentiation of lesser order differential equations whose solutions were known.
- c. Solutions in the form of contour integrals in the complex plane. One independent solution may be expressed as

$$y = \int_a^b \frac{1}{t} e^{(zt + t^3/3 + \lambda^4/t)} dt \quad (2)$$

where the path of integration is such that

$$\oint = t e^{t^3/3 + \lambda^4/t} \Big|_a^b \text{ vanishes.}$$

This formulation was abandoned because of the difficulty of performing the integration and the problem of deriving additional linearly independent solutions.

- d. An analog computer solution was attempted, but was unsuccessful because of instability with respect to initial conditions.

III. Numerical Approximation of Bounds on Eigenvalues

Concurrent with the above investigation, a computer program has been designed whereby the upper bounds of the eigenvalues of Equation (1) may be approximated by the Rayleigh-Ritz method and the lower bounds by the method of Kato,^(14,15) and also by the method of "intermediate problems" originated by Weinstein⁽¹³⁾ and extended by Aronszajn.⁽¹⁶⁾ Use is also made of recent computational forms devised by Bazley⁽¹⁷⁾ and Bazley and Fox.⁽¹⁸⁾ Upper and lower bounds have been calculated as described above and tabulated for a wide range of parameters for the simply supported case. The program for the clamped beam is currently being run, and a program for free-free conditions is in the process of design.

IV. Eigenvalues for the Flat Plate Equation

It can be shown that the Partial Differential Equation for the flat plate may be separated into the same form as for the beam, Equation (1). Thus, the bounds of eigenvalues tabulated for the beam problem may be transformed into natural frequencies for the flat plate.

V. Approximation by Average End Loading

An approximate solution may be obtained by replacing the uniformly distributed load by average fixed end loads (edge loading in the case of the plate). The eigenvalues for this approximation have been tabulated for the simply supported case and are included in the program for the clamped condition. For the simply supported case, the average end load eigenvalues are all greater than the upper bounds calculated from

Rayleigh-Ritz. By comparison with the upper and lower bounds, a quantitative measure of the error using average end loads may be calculated.

VI. Analysis of the Circular Cylinder

Although originally contemplated to be included in the project, an analysis of the eigenvalues of the circular cylinder will be impossible to perform by the completion date of this project. This aspect of the problem involves the eigenvalues of an eighth order differential equation with higher order mixed partials and variable coefficients. Although this problem is vulnerable to attack by the numerical methods described above, time does not permit completion.

VII. Completion of Research Program

Upon completion of the project, it is anticipated that a report will be submitted including the following information in tabular and graphic form:

- a. Upper bounds for the eigenvalues for simply-supported, clamped, and free-free conditions.
- b. Lower bounds for the eigenvalues for all three cases.
- c. Eigenvalues as determined by average end loads, together with upper and lower limits on the error of this approximation.
- d. Formulas to transform tabulated eigenvalues to those for a flat plate.

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